

Spontaneous Breaking of Lorentz Symmetry by Ghost Condensation in Perturbative Quantum Gravity

Mir Faizal

Department of Mathematics, Durham University,
Durham, DH1 3LE, United Kingdom,
faizal.mir@durham.ac.uk

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Abstract

In this paper we will study the spontaneous breakdown of the Lorentz symmetry by ghost condensation in perturbative quantum gravity. Our analysis will be done in the Curci-Ferrari gauge. We will also analyse modification of the BRST and the anti-BRST transformations by the formation of this ghost condensate. It will be shown that even though the modified BRST and the modified anti-BRST transformations are not nilpotent, their nilpotency is restored on-shell.

Key words: Ghost Condensation, Lorentz Symmetry Breaking
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1 Introduction

Lorentz invariance is one of the most important symmetries in nature and has been conformed by all experiments. However, recently the violation of this symmetry is being intensively discussed [1]. Many new experiments are designed to detect any violation of this symmetry [2, 3].

The violation of Lorentz symmetry is being studied because it is expected that interactions in string theories might lead to the spontaneous breakdown of Lorentz symmetry [4, 5]. In fact it is expected that string theory will give spacetime a non-commutative structure and this will in-turn violate Lorentz symmetry [6]. Lorentz symmetry also seems to be violated in many other approaches to quantum gravity [7]. In fact the whole program of Horava-Lifshits theory is based on breaking the Lorentz symmetry so that higher order spatial terms can be added to the classical Lagrangian density without adding any higher order temporal ones [8, 9].

The violation of Lorentz symmetry has also been studied in the context of gravity coupled to Chern-Simons term [10]. Initially this theory was initially studied only at an linearized level [11], however, recently these results have been generalized to include interactions [12]. Dynamical Lorentz symmetry breaking induced by radiative corrections, in a self-interacting fermionic theory has also

been studied [13]. In fact an extension of standard model has been constructed where through the Higgs mechanism, tensor fields acquire non-zero vacuum expectation values and thus break Lorentz symmetry spontaneously [14]. The nature of Nambu-Goldstone bosons associated with Lorentz symmetry breaking have also been thoroughly studied [15]-[17]. Spin-dependent interactions [18] and spin-independent interactions [19] induced by these Nambu-Goldstone bosons have been investigated. However, so far spontaneous breakdown of the Lorentz symmetry has not been studied in the context of perturbative quantum gravity.

In this paper we will investigate the spontaneous violation of Lorentz symmetry induced by ghost condensation in perturbative quantum gravity. Ghost condensation in Yang-Mills theories has been studied in the Curci-Ferrari gauge [20], which is obtained by the inclusion of non-linear terms to the usual Faddeev-Popov Lagrangian density [21]-[25]. Ghost condensation has also been studied in the context of ghosts associated with higher derivatives, which occur in theories of modified gravity [26]-[29]. However, so far no work has been done on ghost condensation of the Faddeev-Popov ghosts in Curci-Ferrari gauge in perturbative quantum gravity. The BRST and the anti-BRST symmetries for perturbative quantum gravity in linear gauges have been studied by a number of authors [30]-[32] and their work has been summarized by N. Nakanishi and I. Ojima [33]. The BRST and anti-BRST symmetries for perturbative quantum gravity in Curci-Ferrari gauge have also been recently studied [34]. In this paper we will study ghost condensation and its consequences for perturbative quantum gravity in Curci-Ferrari gauge.

It may be noted that it is not possible to perform an explicit violation of Lorentz symmetry as this will be incompatible with Bianchi identities and the covariant conservation laws for the energy-momentum and spin-density tensors, whereas spontaneous Lorentz breaking evades this difficulty [35]. So there will be no fundamental change in the classical action of this theory. In this way this present work is slightly different from the work on Horava-Lifshits gravity which is based on modifying the classical theory.

2 BRST and Anti-BRST Symmetries

In this section we will review usual the BRST and the anti-BRST symmetry for perturbative quantum gravity [34]. The Lagrangian density for pure gravity with cosmological constant λ is given by

$$\mathcal{L} = \sqrt{-g}(R - 2\lambda), \quad (1)$$

where we have adopted units, such that $16\pi G = 1$. In perturbative gravity one splits the full metric g_{ab} into the metric for the background flat spacetime η_{ab} and a small perturbation around it being h_{ab} . The covariant derivatives along with the lowering and raising of indices are compatible with the metric for the background spacetime and small perturbation h_{ab} is viewed as the field that is to be quantized.

All the degrees of freedom in h_{ab} are not physical as the Lagrangian density for it is invariant under a gauge transformation,

$$\delta_\Lambda h_{ab} = D_{ab}^e \Lambda_e$$

$$\begin{aligned}
&= [\delta_b^e \partial_a + \delta_a^e \partial_b + \eta^{ce} (\partial_c h_{ab}) + \\
&\quad \eta^{ec} h_{ac} \partial_b + \eta^{ec} h_{cb} \partial_a] \Lambda_e,
\end{aligned} \tag{2}$$

where Λ^a is a vector field. These unphysical degrees of freedom give rise to constraints [36] in the canonical quantization and divergences in the partition function [37] in the path integral quantization. So before we can quantize this theory, we need to fix a gauge. This is achieved by addition of a ghost term and a gauge fixing term to the classical Lagrangian density. Now let us denote the sum of a ghost term and a gauge fixing term by \mathcal{L}_g , which is given by

$$\begin{aligned}
\mathcal{L}_g &= -\frac{i}{2} s \bar{s} [h^{ab} h_{ab}] + \frac{i\alpha}{2} \bar{s} [b^a c_a] \\
&= \frac{i}{2} \bar{s} s [h^{ab} h_{ab}] - \frac{i\alpha}{2} s [b^a \bar{c}_a],
\end{aligned} \tag{3}$$

where the BRST transformations are given by

$$\begin{aligned}
s h_{ab} &= D_{ab}^e c_e, \\
s c^a &= -c_b \partial^b c^a, \\
s \bar{c}^a &= b^a, \\
s b^a &= 0,
\end{aligned} \tag{4}$$

and the anti-BRST transformations are given by

$$\begin{aligned}
\bar{s} h_{ab} &= D_{ab}^e \bar{c}_e, \\
\bar{s} c^a &= -b^a - 2\bar{c}_b \partial^b c^a, \\
\bar{s} \bar{c}^a &= -\bar{c}_b \partial^b \bar{c}^a, \\
\bar{s} b^a &= -b^b \partial_b c^a.
\end{aligned} \tag{5}$$

In the next section we will analyse perturbative quantum gravity in the Curci-Ferrari gauge.

3 Lorentz Symmetry Breaking

In order to study spontaneous breaking of Lorentz symmetry, we have to modify the above mentioned BRST and anti-BRST transformations by the addition of non-linear terms to them. Thus the modified BRST transformations are given by

$$\begin{aligned}
s h_{ab} &= D_{ab}^e c_e, \\
s c^a &= -c_b \partial^b c^a, \\
s \bar{c}^a &= b^a - \bar{c}^b \partial_b c^a, \\
s b^a &= -b^b \partial_b c^a - \bar{c}^b \partial_b c^d \partial_d c^a,
\end{aligned} \tag{6}$$

and the modified anti-BRST transformations are given by

$$\begin{aligned}
\bar{s} h_{ab} &= D_{ab}^e \bar{c}_e, \\
\bar{s} \bar{c}^a &= -\bar{c}_b \partial^b \bar{c}^a, \\
\bar{s} c^a &= -b^a - \bar{c}^b \partial_b c^a, \\
\bar{s} b^a &= -b^b \partial_b \bar{c}^a + c^b \partial_b \bar{c}^d \partial_d \bar{c}^a.
\end{aligned} \tag{7}$$

Now as we have modified the BRST and the anti-BRST transformations, so the ghost term also gets modified. The sum of this modified ghost term and the gauge fixing term, is given by

$$\begin{aligned}\mathcal{L}_g^{(mod)} &= \frac{i}{2} s\bar{s} [h^{ab}h_{ab} - i\alpha\bar{c}^a c_a] \\ &= -\frac{i}{2} \bar{s}s [h^{ab}h_{ab} - i\alpha\bar{c}^a c_a].\end{aligned}\quad (8)$$

It may be noted that just like the Yang-Mills theories in Curci-Ferrari gauge, the perturbative quantum gravity also possess a double BRST symmetry, where the gauge fixing term and the modified ghost term is written as a combination of the BRST and the anti-BRST transformations. This Lagrangian density for the sum of the modified ghost and gauge fixing terms in the Curci-Ferrari gauge is related to the usual Lagrangian density for the ghost and gauge fixing terms as follows,

$$\mathcal{L}_g^{(mod)} = \mathcal{L}_g + \frac{\alpha}{2} \bar{c}^b \partial_b c^a \cdot \bar{c}^c \partial_c c_a. \quad (9)$$

Thus apart from the usual Lagrangian density there is a non-linear term in it. However, we can linearise this non-linear term by means of Hubbard-Stratonovich transformations, as follows

$$\frac{\alpha}{2} \bar{c}^b \partial_b c^a \cdot \bar{c}^c \partial_c c_a = -\frac{1}{2\alpha} \phi^a \phi_a - i\phi^a \bar{c}^b \partial_b c_a. \quad (10)$$

The field ϕ^a introduced here has a vanishing ghost number and is required to be hermitian to maintain the hermiticity of the total Lagrangian density. Thus after using the Hubbard-Stratonovich transformations, the Lagrangian density for the sum of the modified ghost and the gauge fixing terms in the Curci-Ferrari gauge becomes,

$$\mathcal{L}_g^{(mod)} = L_{gf} + i\bar{c}^a N_{ab} c^b - \frac{1}{2\alpha} \phi^a \phi_a, \quad (11)$$

where

$$N_{ab} = K_{ab} - \phi_a \partial_b, \quad (12)$$

here K_{ab} is the contribution coming from the original ghost term and is given by

$$K_{ab} = \eta_{ae} \eta_{bf} \eta^{nm} \eta^{pq} D_{np}^e D_{mq}^f. \quad (13)$$

Now we sum over all one-loop ghost diagrams with arbitrary number of external ϕ^a fields. This gives us an effective potential $V[\phi]$, which is given by

$$\int d^4x V[\phi] = \int d^4x \frac{1}{2\alpha} \phi^a \phi_a + i \log [\det(N_{ab})]. \quad (14)$$

This effective potential obtained from Eq. (14) is divergent and thus has to be regulated. The renormalized effective potential thus obtained, is given by

$$V[\phi] = \phi^a \phi_a \left[\frac{1}{2\alpha} + \frac{1}{32\pi^2} \left(\log \left(\frac{|\phi|}{4\pi\mu^2} \right) + C \right) \right] \quad (15)$$

The stationary point for this effective potential is given by

$$\frac{\delta V[\phi]}{\delta \phi^a} = 0. \quad (16)$$

The Eq. (16) apart from having the trivial solution $\phi^a = 0$, also has the non-trivial solutions $\phi^a = \pm \nu^a$. In semi-classical approximation, the field ϕ^a is shifted as follows

$$\phi^a \rightarrow \phi_{(cl)}^a + \tilde{\phi}^a, \quad (17)$$

where $\phi_{(cl)}^a$ is the classical field and $\tilde{\phi}^a$ represents the quantum fluctuations to it. The vacuum expectation value of the field ϕ^a is required to coincide with the classical field so that the vacuum expectation value of the quantum fluctuations vanish. Now in the non-trivial vacuum $\phi^a = \pm \nu^a$, we get a non-vanishing vacuum expecting value for the vector field $\phi_{(cl)}^a$, and this spontaneously breaks the Lorentz symmetry,

$$\phi_{(cl)}^a = \langle \phi^a \rangle = \pm \nu^a. \quad (18)$$

In this section we showed that the formation of ghost condensate in perturbative quantum gravity spontaneously break the Lorentz symmetry. In the next section we will investigate the BRST and anti-BRST symmetries in this phase, where the Lorentz symmetry is spontaneously broken.

4 Modified BRST and anti-BRST Transformations

The formation of ghost condensate not only spontaneously breaks the Lorentz symmetry but it also spoils the nilpotency of the BRST and the anti-BRST transformations. However, we will see in this section that the nilpotency of these modified BRST and modified anti-BRST transformations is restored on-shell. The BRST transformations get modified due to the formation of ghost condensates as follows,

$$\begin{aligned} s h_{ab} &= D_{ab}^e c_e, \\ s c^a &= -c_b \partial^b c^a, \\ s \bar{c}^a &= b^a - \bar{c}^b \partial_b c^a, \\ s b^a &= -b^b \partial_b c^a, \\ s \phi^a &= 2\psi^a, \\ s \psi^a &= -\frac{1}{2} \psi^b \partial_b \psi^a, \\ s \bar{\psi}^a &= \phi^a - i \bar{c}^b \partial_b c^a, \end{aligned} \quad (19)$$

and the anti-BRST transformations get modified as follows,

$$\begin{aligned} \bar{s} h_{ab} &= D_{ab}^e \bar{c}_e, \\ \bar{s} \bar{c}^a &= -\bar{c}_b \partial^b \bar{c}^a, \\ \bar{s} c^a &= -b^a - \bar{c}^b \partial_b c^a, \\ \bar{s} b^a &= -b^b \partial_b \bar{c}^a, \\ \bar{s} \phi^a &= 2\bar{\psi}^a, \end{aligned}$$

$$\begin{aligned}
\bar{s}\psi^a &= -\frac{1}{2}\bar{\psi}^b\partial_b\bar{\psi}^a, \\
\bar{s}\bar{\psi}^a &= \phi^a - i\bar{c}^b\partial_b c^a.
\end{aligned}
\tag{20}$$

Here ϕ^a plays the role of a new Nakanishi-Lautrup field and ψ^a and $\bar{\psi}^a$ play the role of new ghosts and anti-ghosts respectively. These new BRST and anti-BRST transformations are not nilpotent, because

$$\begin{aligned}
s^2\bar{\psi}^a &= 2\psi^a - b^b\partial_b c^a \neq 0, \\
\bar{s}^2\psi^a &= 2\bar{\psi}^a + \bar{b}^b\partial_b \bar{c}^a \neq 0.
\end{aligned}
\tag{21}$$

However, their nilpotency is restored by using the equation of motion for these fields and thus on-shell version of the above two transformations is given by

$$\begin{aligned}
[s^2\bar{\psi}^a]_{on-shell} &= 0, \\
[\bar{s}^2\psi^a]_{on-shell} &= 0.
\end{aligned}
\tag{22}$$

The sum of the gauge fixing term and the ghost term also gets modified because of the modification of the BRST and the anti-BRST transformations. However, even after this modification the sum of the gauge fixing term and the ghost term possess a double BRST symmetry on-shell and thus can be written as a combination of the BRST and the anti-BRST transformations on-shell,

$$\begin{aligned}
\mathcal{L}_g^{(new)} &= \frac{i}{2}s\bar{s}\left[h^{ab}h_{ab} - i\alpha\bar{c}^a c_a - i\frac{\alpha}{2}\phi^a\phi_a\right] \\
&= -\frac{i}{2}\bar{s}s\left[h^{ab}h_{ab} - i\alpha\bar{c}^a c_a - i\frac{\alpha}{2}\phi^a\phi_a\right].
\end{aligned}
\tag{23}$$

The sum of the gauge fixing term and modified ghost term is related to the sum of the usual ghost term and gauge fixing term as follows,

$$\mathcal{L}_g^{(new)} = \mathcal{L}_g - i\alpha\phi^a\bar{c}^b\partial_b c_a - \alpha\bar{\psi}^a\psi_a + \frac{\alpha}{2}\phi^a\phi_a.
\tag{24}$$

It may be noted that the appearance of the term $\alpha\bar{\psi}^a\psi_a$ only produces a multiplicative overall factor, which can be absorbed in the normalisation constant of the partition function.

5 Conclusion

We have seen how Lorentz symmetry is spontaneously broken by the formation of ghost condensates in perturbative quantum gravity. We have also analysed the modification of the BRST and the anti-BRST transformations by this ghost condensation. We have shown that even though the modified BRST and the modified anti-BRST transformations are not nilpotent, their nilpotency is restored on-shell.

One of the possible signals of Lorentz violation may come from CMB [38] and other high-energy astrophysical observations [39]. Violation of Lorentz might be helpful in explaining the polarization of CMB.

Lorentz violation will also have interesting phenomenological signatures. In fact if there is a non-vanishing vacuum expecting value for the vector field then

it might lead to a decrease in the anomaly frequency of a positron if the anomaly frequency of an electron is increased [40]. However, so far no violation of Lorentz symmetry has been detected [42, 43]. Another signature of Lorentz symmetry breaking might come from the spectral analysis of the spectra of atoms made up of matter and similar atoms made up of anti-matter. In fact calculations on the spectrum of hydrogen and anti-hydrogen show that tiny differences will occur in some lines, and no differences will occur in others if the Lorentz symmetry is spontaneously broken [41].

The non-vanishing vacuum expecting value for the vector field could possible explain the occurrence of the cosmological constant. The fact that spontaneous breakdown of the Lorentz symmetry can only occur at very high energies might explain why the cosmological constant has such a small value.

Our work has been done in flat spacetime and it will be interesting to generalise this to general spacetimes or at least maximally symmetric spacetimes like the de Sitter spacetime and anti-de Sitter spacetime. A similar analysis might lead to a spontaneous breakdown of the de Sitter or anti-de Sitter invariance in those spacetimes.

References

- [1] V. A. Kostelecky, Phys. Rev. **D** 69, 105009 (2003)
- [2] A. Kostelecky and N. Russell, Rev. Mod. Phys. 83, 11 (2011)
- [3] V. A. Kostelecky, ed., CPT and Lorentz Symmetry III, World Scientific, Singapore, (2005)
- [4] V.A. Kostelecky and S. Samuel, Phys. Rev. **D** 39, 683 (1989)
- [5] D. Colladay and V. A. Kostelecky, Phys. Rev. **D** 55, 6760 (1997)
- [6] N. Seiberg and E. Witten, **JHEP**. 9909, 032 (1999)
- [7] R. Bluhm, Breaking Lorentz symmetry, Phys. World. (2004)
- [8] P. Horava, **JHEP**. 0903, 020 (2009)
- [9] P. Horava, Phys. Rev. Lett. 102, 161301 (2009)
- [10] R. Jackiw, S. Y. Pi, Phys. Rev. D 68, 104012 (2003)
- [11] T. Mariz, J. R. Nascimento, E. Passos and R. F. Ribeiro, Phys. Rev. **D** 70, 024014 (2004)
- [12] T. Mariz, J. R. Nascimento, A. Yu. Petrov, L. Y. Santos, A. J. da Silva, Phys. Lett. **B** 661, 312 (2008)
- [13] M. Gomes, T. Mariz, J. R. Nascimento and A. J. da Silva, Phys. Rev. **D** 77, 105002 (2008)
- [14] D. Colladay and V. A. Kostelecky, Phys. Rev. **D** 55, 6760 (1997)
- [15] R. Bluhm and V.A. Kostelecky, Phys. Rev. **D** 71, 065008 (2005)
- [16] V. A. Kostelecky and R. Potting, Phys. Rev. **D** 79, 065018 (2009)

- [17] V. A. Kostelecky and R. Potting, Gen. Rel. Grav. **37**, 1675 (2005)
- [18] N. Arkani-Hamed, H. C. Cheng, M. Luty, and J. Thaler, **JHEP**. 0507, 029 (2005)
- [19] V.A. Kostelecky and J. Tasson, Phys. Rev. Lett. **102**, 010402 (2009)
- [20] G. Curci and R. Ferrari, Nuovo Cim. **A 32**, 151 (1976)
- [21] K. I. Kondo and T. Shinohara, Phys. Lett. **B491**, 263 (2000)
- [22] M. A. L. Capri, V. E. R. Lemes, R.F. Sobreiro, S. P. Sorella, R. Thibes, Phys. Rev. **D 77**, 105023 (2008)
- [23] M. A. L. Capri, D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, H. Verschelde, Phys. Rev. **D 73** 014001 (2006)
- [24] A. R. Fazio, Mod. Phys. Lett. **A 20** 585 (2005)
- [25] K.-I. Kondo, Phys. Lett. **B 572** 210 (2003)
- [26] B. Feldstein, Phys. Rev. **D78**, 064061 (2008)
- [27] S. Mukohyama, JCAP0610, 011 (2006)
- [28] S. Mukohyama, Phys.Rev. **D 71**, 104019 (2005)
- [29] Nima Arkani-Hamed, Hsin-Chia Cheng, Markus A. Luty, Shinji Mukohyama, JHEP, 0405, 074 (2004)
- [30] N. Nakanishi, Prog. Theor. Phys. **59**, 972 (1978)
- [31] T. Kugo and I. Ojima, Nucl. Phys. **B 144**, 234 (1978)
- [32] K. Nishijima and M. Okawa, Prog. Theor. Phys. **60**, 272 (1978)
- [33] N. Nakanishi and I. Ojima, Covariant operator formalism of gauge theories and quantum gravity - World Sci. Lect. Notes. Phys - (1990)
- [34] M. Faizal, Found. Phys. **41**, 270 (2011)
- [35] V. A. Kostelecky, Phys. Rev. **D 69**, 105009 (2004)
- [36] M. Henneaux and C. Teitelboim, Quantization of Gauge Systems. Princeton University Press,- (1992)
- [37] J. T. Mieg, J. Math. Phys. **21**, 2834 (1980)
- [38] Yi-Fu Cai, Mingzhe Li and Xinmin Zhang, **JCAP**. 1001, 017 (2009)
- [39] T. Jacobson, S. Liberati and David Mattingly, Annals. Phys. **321**, 150 (2006)
- [40] R. Bluhm, V. A. Kostelecky and N. Russell, Phys. Rev. **D 57**, 3932 (1998)
- [41] R. Bluhm, V. A. Kostelecky and N. Russell, Phys. Rev. Lett. **82**, 2254 (1999)
- [42] R. K. Mittleman, I. I. Ioannou, H. G Dehmelt, and N. Russell, Phys. Rev. Lett. **83**, 2116 (1999)
- [43] H. G. Dehmelt and R. K. Jr. Van Dyck, Phys. Rev. Lett. **83**, 4694 (1999)